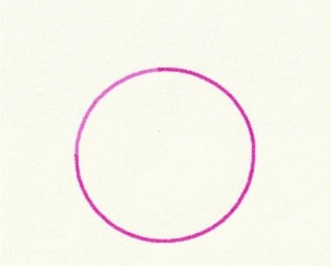
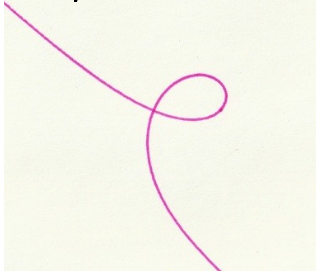
A pure angular-based coordinate system and curves resulting from a linear relation between two angles α and β in two dimensions



$\beta = 0.5\,\alpha$

$\beta = 2\,\alpha + 95°$



Version 1 from 9 August 2026

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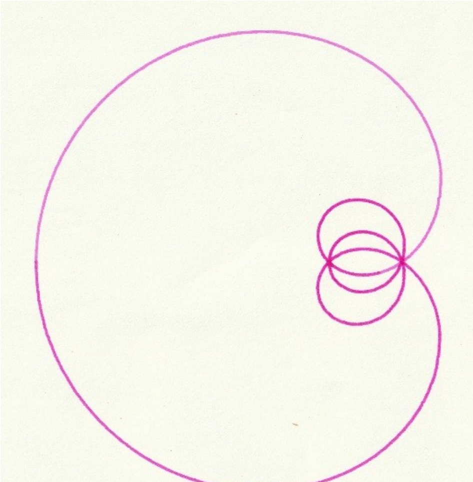


$\beta = 0.8\,\alpha$

This paper in form of a presentation comprises 36 slides or pages, a content overview, and can be downloaded as pdf via the following link (file size about 5 MB):

https://novam-research.com/resources/A-pure-angular-based-coordinate-system-and-resulting-curves-from-a-linear-relation-between-beta-and-alpha-in-2D.pdf

This paper in form of a presentation was prepared by using LibreOffice Impress.

Content overview

1 Introduction and an example of a definition of two angles β and α in two dimensions ...

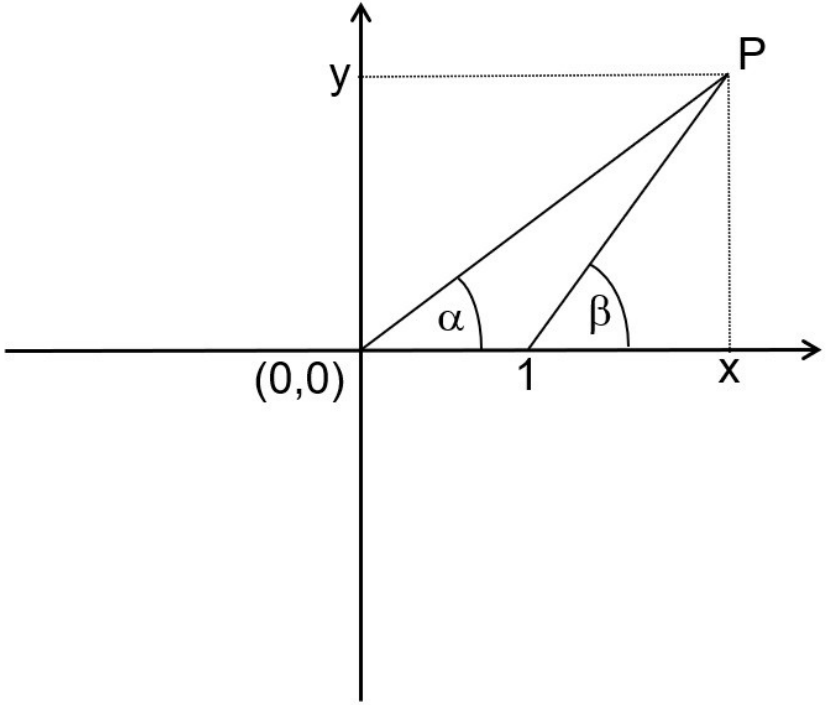
In science, physics, engineering, mathematics, and geometry coordinate systems play often an important role. Examples are Cartesian coordinates, polar coordinates, spherical coordinates, or cylindrical coordinates. In these coordinate systems the position of a point P (or an extended object) is described by at least one spatial distance coordinate and the other type of coordinates – if any – are angular coordinates.

Initially, around 1988, the author of this paper / presentation considered the possibility of coordinate systems which are based exclusively on angular coordinates and if they could have a special meaning. One of the associated questions was, for the simplest case of two spatial dimensions, what type of curves arise when a point P = (α,β) traverses a sequence of positions which is given by the linear function or relationship

$$\beta = V\alpha + W$$

where V and W are parameters.

We consider here this example of a definition of the angular coordinates β and α within a Cartesian coordinate system which is used in this work. The conversion of (α,β) into (x,y) is especially useful for plotting curves $\beta = \beta(\alpha)$



Point P = $(\alpha,\beta) = (x,y)$

Definition of the angles α and β and their conversion into (x,y) of a cartesian coordinate system

$$\tan(\alpha) = y / x$$

$$\tan(\beta) = y / (x - 1)$$

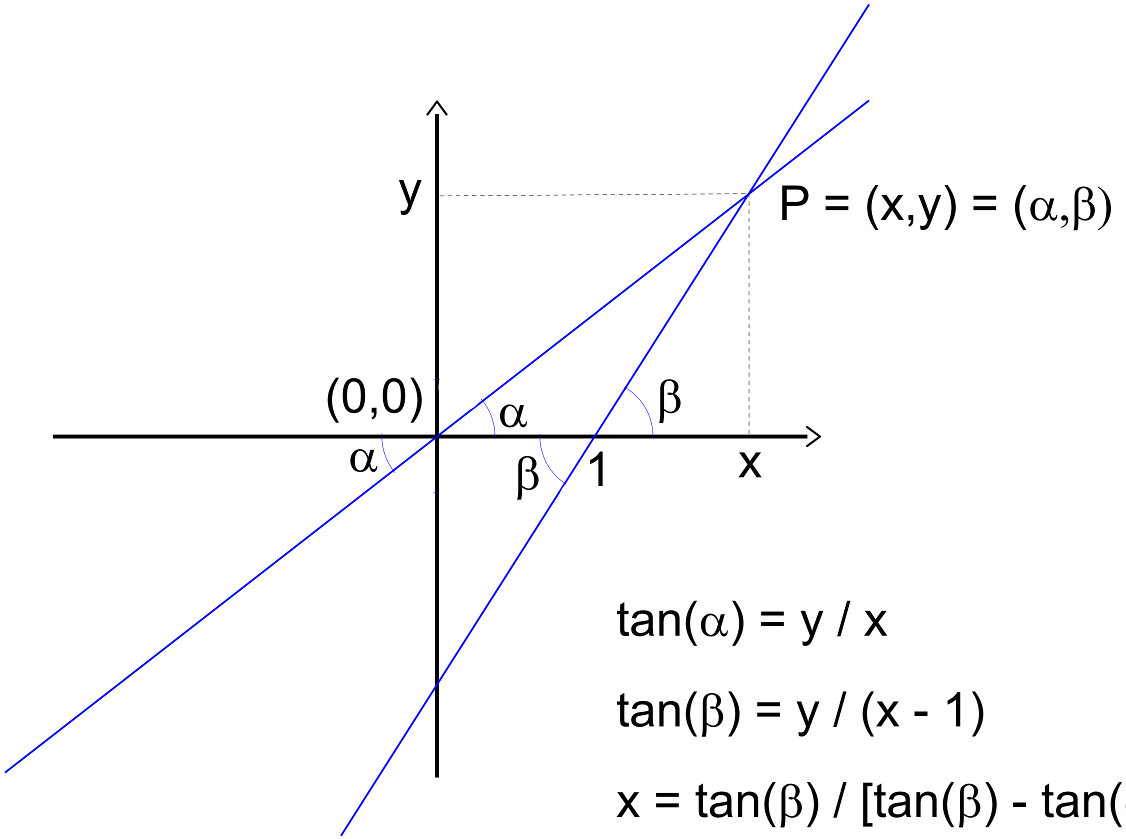
$$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)]$$

$$y = x \tan(\alpha)$$

This is another sketch of the same definition of the angular coordinates β and α within a Cartesian coordinate system.

It illustrates that the point P emerges from an intersection of two lines or rays. One line or ray can rotate by an angle α about a pivot at $(x,y) = (0,0)$ and the other line or ray can rotate by an angle β about a pivot at $(x,y) = (1,0)$.

This example of a definition of β and α is used in this work.



$\tan(\alpha) = y / x$

$\tan(\beta) = y / (x - 1)$

$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)]$

$y = x \tan(\alpha)$

In general β could be any function of α , i.e.

$\beta = \beta(\alpha)$

or in the form of a parameter representation

$\alpha = \alpha(t)$

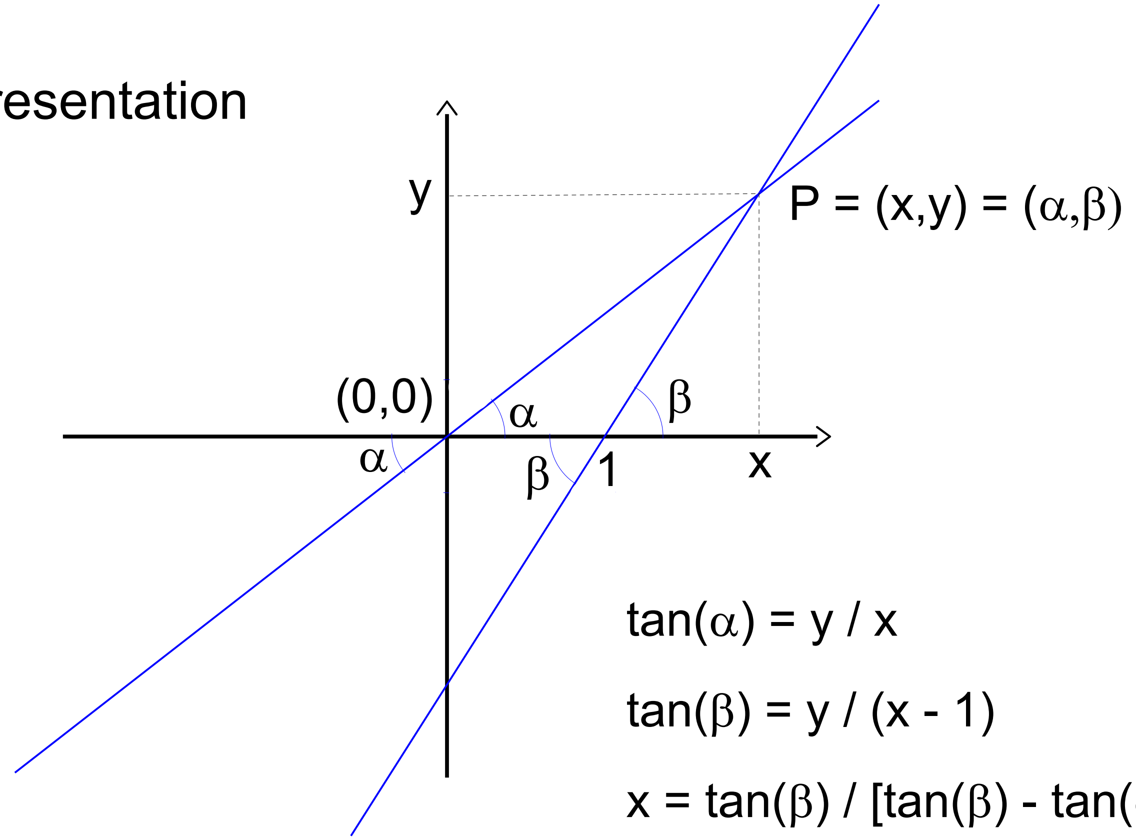
$\beta = \beta(t)$

whereby t is a progressive parameter. In physics that could be for example the time.

In this work we consider only a linear relationship between β and α in the form of

$\beta = V\alpha + W$

where V and W are parameters



$\tan(\alpha) = y / x$

$\tan(\beta) = y / (x - 1)$

$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)]$

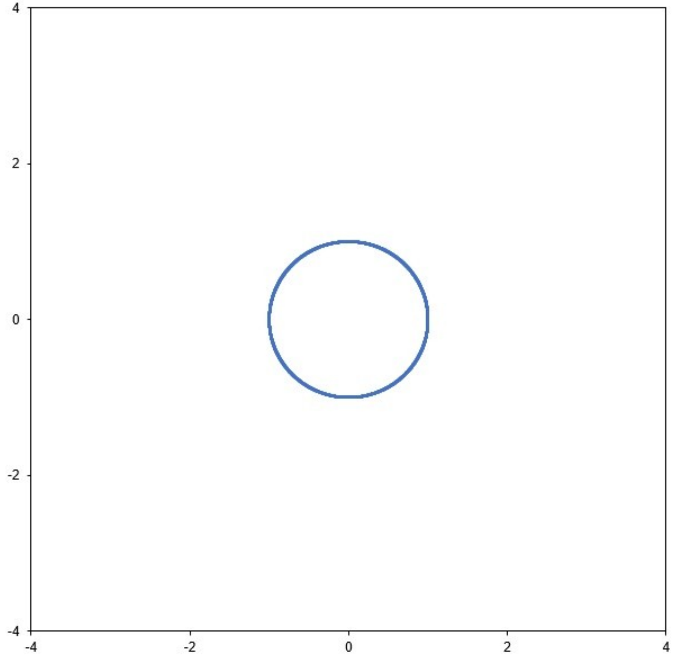
$y = x \tan(\alpha)$

2 Examples of curves generated by a linear relation or function $\beta = V\alpha + W$ with parameters V and W ...

2 a Examples of $\beta = V\alpha + W$ curves which are created with Excel by using the chart (plot) type "x y scatter" ...

If the chart (plot) type "scatter with smooth lines" is selected, then the curve is often overlayed by a few or many straight lines. They may arise by a line which connects two dots $P_{n+2} = (x_{n+2}\ , y_{n+2})$ and $P_n = (x_n\ , y_n)$ of the curve when the dot P_{n+1} corresponds to an infinity position $\beta = \alpha$

$\beta = 0.5\,\alpha$



$\alpha_{n+1} = \alpha_n + (2\pi N / 2000)$ $\quad n = 0, 1, 2, 3, \ldots, 2000$

N = Number of revolutions in units of 2π	=	1
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

V = 0.5
W = 0

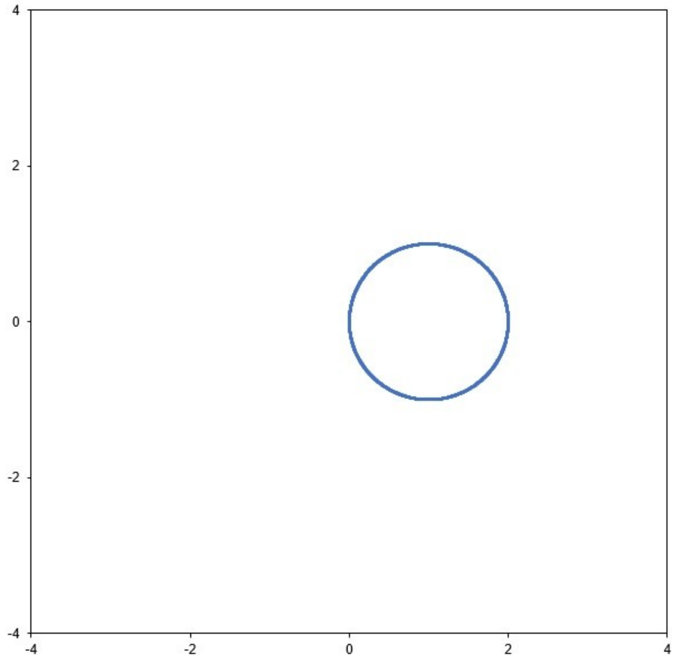
$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)]$ $\qquad y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00314	0.00157	-1.00000	-0.00314
0.00628	0.00314	-0.99998	-0.00628
0.00942	0.00471	-0.99996	-0.00942
0.01257	0.00628	-0.99992	-0.01257
0.01571	0.00785	-0.99988	-0.01571

This W = 0 type curve on this and the following page (both a circle) indicate that $V \rightarrow V' = 1 / V$ results in the same type of curve but they spatially shifted

$\beta = 2\,\alpha$



$\alpha_{n+1} = \alpha_n + (2\pi N / 2000) \qquad n = 0 , 1 , 2 , 3, \dots , 2000$

N = Number of revolutions in units of 2π	=	1
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

V = 2
W = 0

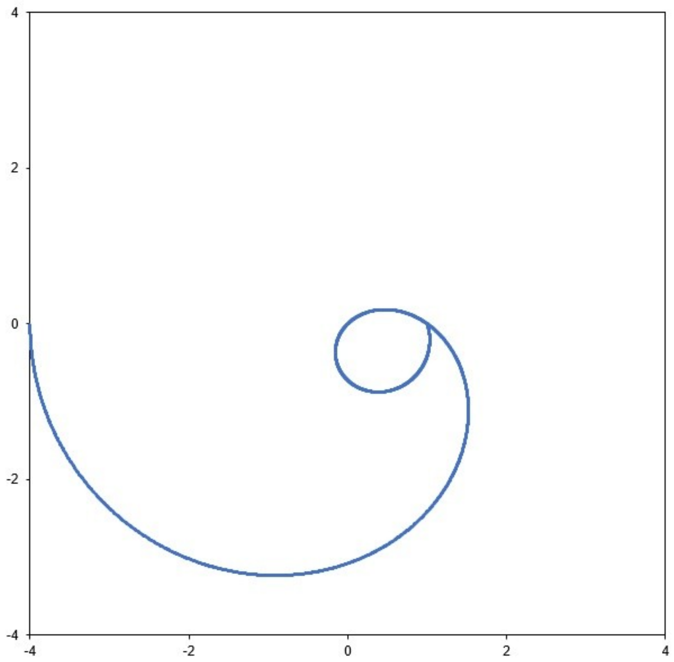
$x = \tan(\beta) / [\, \tan(\beta) - \tan(\alpha) \,] \qquad y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00314	0.00628	1.99998	0.00628
0.00628	0.01257	1.99992	0.01257
0.00942	0.01885	1.99982	0.01885
0.01257	0.02513	1.99968	0.02513
0.01571	0.03142	1.99951	0.03141

This W = 0 type curve on this and the previous page (both a circle) indicate that $V \rightarrow V' = 1 / V$ results in the same type of curve but they spatially shifted

$\beta = 0.8\ \alpha$ Number of revolutions of α (in units of 2π) = 1



$\alpha_{n+1} = \alpha_n + (2\pi N / 2000)$ $n = 0, 1, 2, 3, \ldots, 2000$

N = Number of revolutions in units of 2π	=	1
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

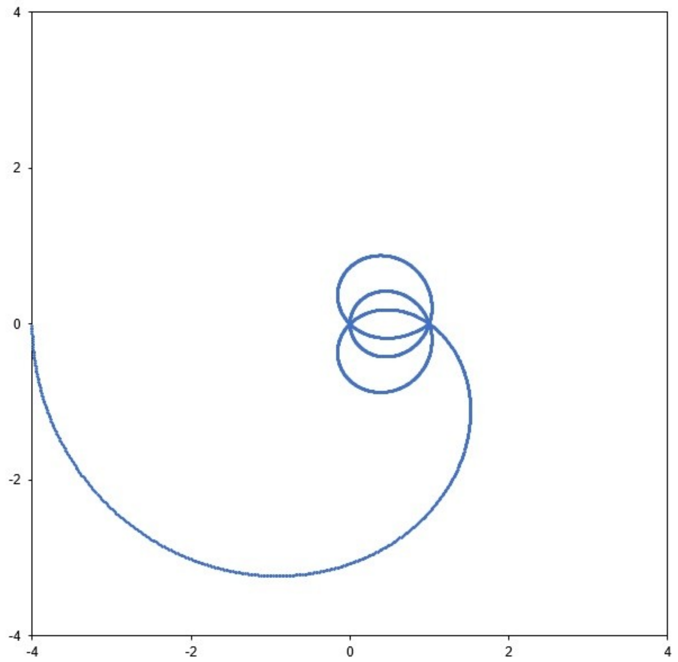
V = 0.8
W = 0

$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)]$ $y = x \tan(\alpha)$

Excerpt of the table α, β, x, y

α	β	x	y
0.00000			
0.00314	0.00251	-3.99998	-0.01257
0.00628	0.00503	-3.99991	-0.02513
0.00942	0.00754	-3.99979	-0.03770
0.01257	0.01005	-3.99962	-0.05026
0.01571	0.01257	-3.99941	-0.06283

$\beta = 0.8\ \alpha$ Number of revolutions of α (in units of 2π) = 2



$\alpha_{n+1} = \alpha_n + (2\pi\ N / 2000)$ $n = 0, 1, 2, 3, \ldots, 2000$

N = Number of revolutions in units of 2π	=	2
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

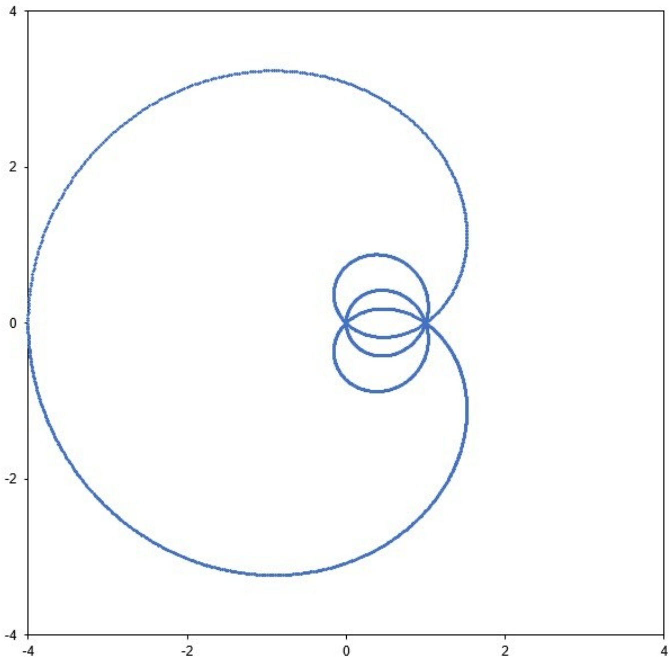
V = 0.8
W = 0

$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)]$ $y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00628	0.00503	-3.99991	-0.02513
0.01257	0.01005	-3.99962	-0.05026
0.01885	0.01508	-3.99915	-0.07539
0.02513	0.02011	-3.99848	-0.10051
0.03142	0.02513	-3.99763	-0.12563

$\beta = 0.8\,\alpha$ Number of revolutions of α (in units of 2π) = 3



$\alpha_{n+1} = \alpha_n + (2\pi N / 2000) \qquad n = 0 , 1 , 2 , 3, \ldots , 2000$

N = Number of revolutions in units of 2π	=	3
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

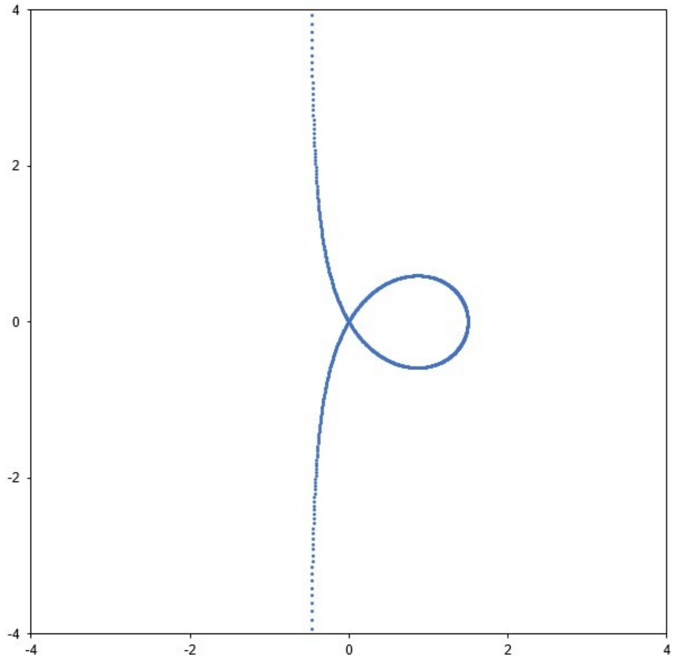
V = 0.8
W = 0

$x = \tan(\beta) / [\tan(\beta) - \tan(\alpha)] \qquad y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00942	0.00754	-3.99979	-0.03770
0.01885	0.01508	-3.99915	-0.07539
0.02827	0.02262	-3.99808	-0.11307
0.03770	0.03016	-3.99659	-0.15074
0.04712	0.03770	-3.99467	-0.18838

$\beta = 3\,\alpha$



$\alpha_{n+1} = \alpha_n + (2\pi\, N / 2000)$ $\quad n = 0 , 1 , 2 , 3, \dots , 2000$

N = Number of revolutions in units of 2π	=	1
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

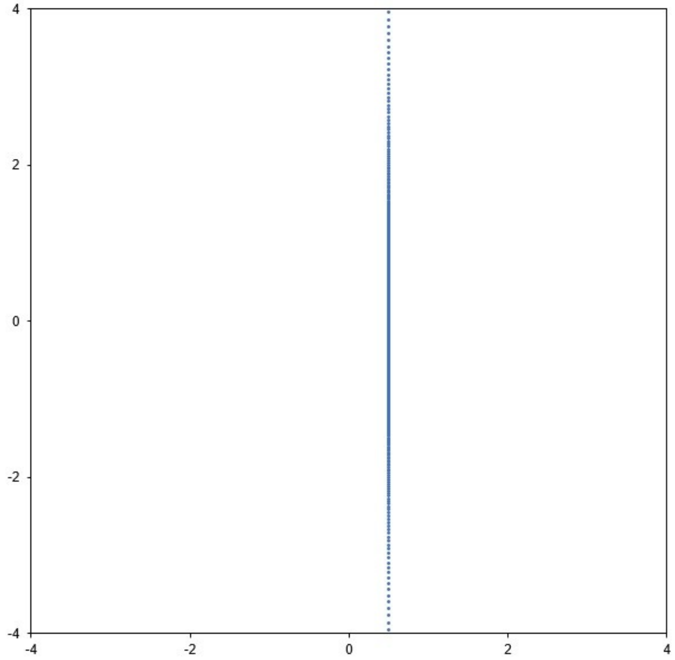
V = 3
W = 0

$x = \tan(\beta) / [\, \tan(\beta) - \tan(\alpha)\,]$ $\quad y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00314	0.00942	1.49998	0.00471
0.00628	0.01885	1.49992	0.00942
0.00942	0.02827	1.49982	0.01414
0.01257	0.03770	1.49968	0.01885
0.01571	0.04712	1.49951	0.02356

β = -1 α



$\alpha_{n+1} = \alpha_n + (2\pi\ N / 2000)$ n = 0 , 1 , 2 , 3, ... , 2000

N = Number of revolutions in units of 2π	=	1
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

V = -1
W = 0

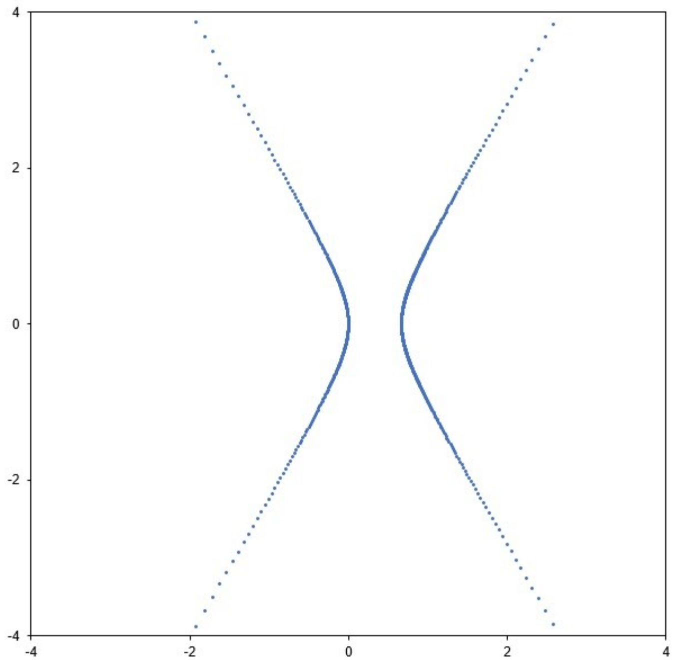
$x = \tan(\beta) / [\ \tan(\beta) - \tan(\alpha)\ ]$ $y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00314	-0.00314	0.50000	0.00157
0.00628	-0.00628	0.50000	0.00314
0.00942	-0.00942	0.50000	0.00471
0.01257	-0.01257	0.50000	0.00628
0.01571	-0.01571	0.50000	0.00785

This curve is a straight line parallel to the y axis

$\beta = -2\,\alpha$



$\alpha_{n+1} = \alpha_n + (2\pi\ N / 2000)$ n = 0 , 1 , 2 , 3, ... , 2000

N = Number of revolutions in units of 2π	=	1
C = Fixed number of generated dots	=	2000

$\beta = V\alpha + W$

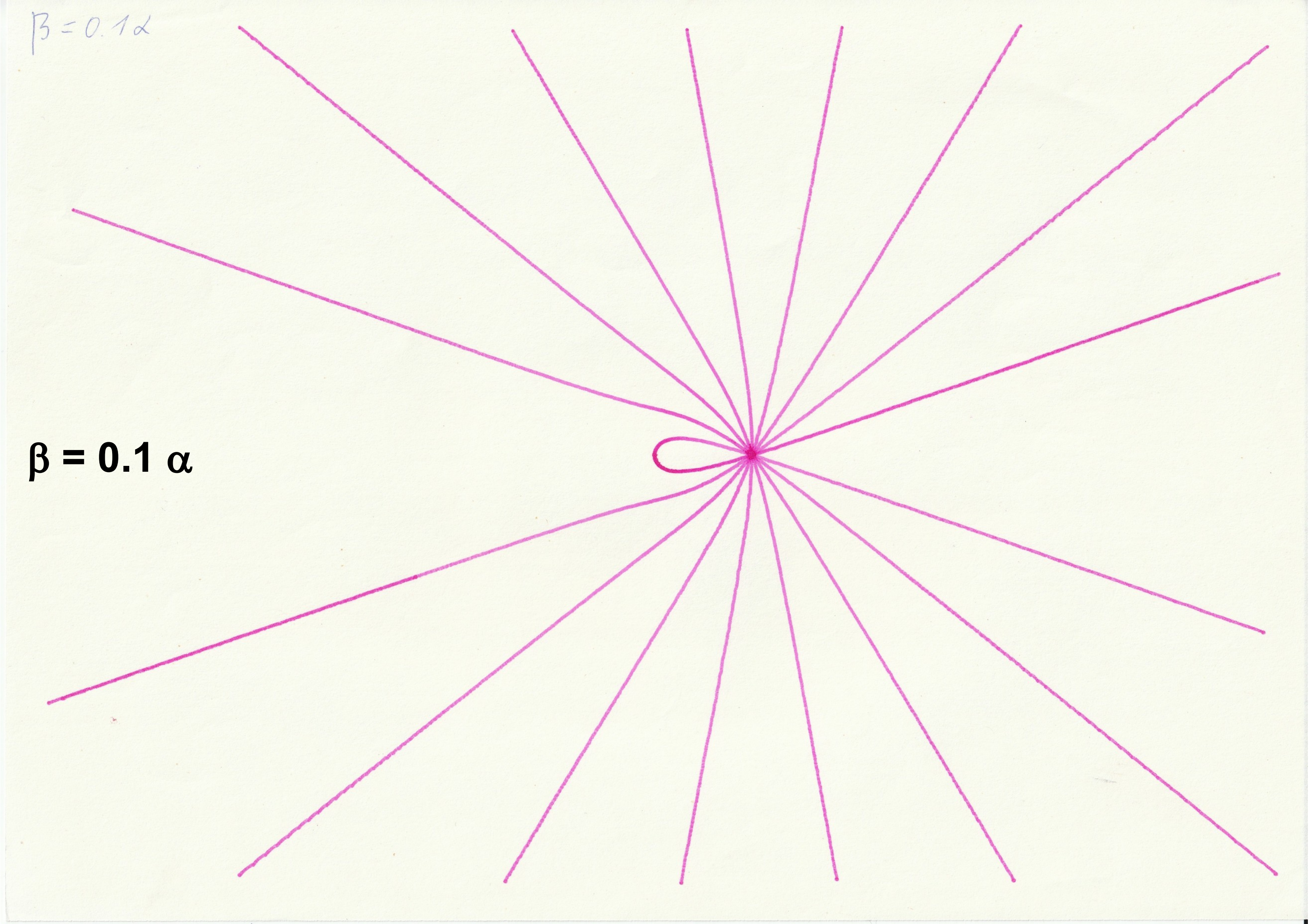
V = -2
W = 0

$x = \tan(\beta) / [\ \tan(\beta) - \tan(\alpha)\ ]$ $y = x \tan(\alpha)$

Excerpt of the table α , β , x , y

α	β	x	y
0.00000			
0.00314	-0.00628	0.66667	0.00209
0.00628	-0.01257	0.66668	0.00419
0.00942	-0.01885	0.66669	0.00628
0.01257	-0.02513	0.66670	0.00838
0.01571	-0.03142	0.66672	0.01047

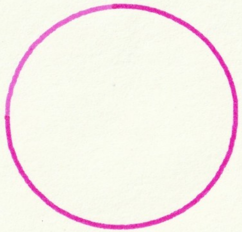
2 b Scanned images of some examples of $\beta = V\alpha + W$ curves which were created around 1988 by a computer program and a classical plotter on A4 type paper ...



$\beta = 0.1\ \alpha$

$\beta = 0.5\ \alpha$

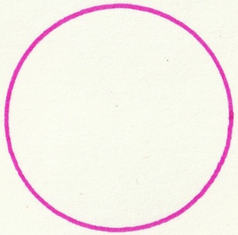
$\beta = 0.5\ \alpha$



This W = 0 type curve on this and the following page (both a circle) indicate that $V \rightarrow V' = 1 / V$ results in the same type of curve but they are spatially shifted

$\beta = 2\alpha$

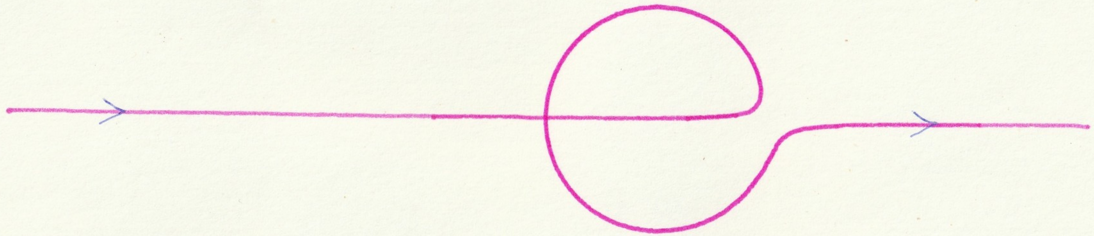
$\beta = 2\ \alpha$



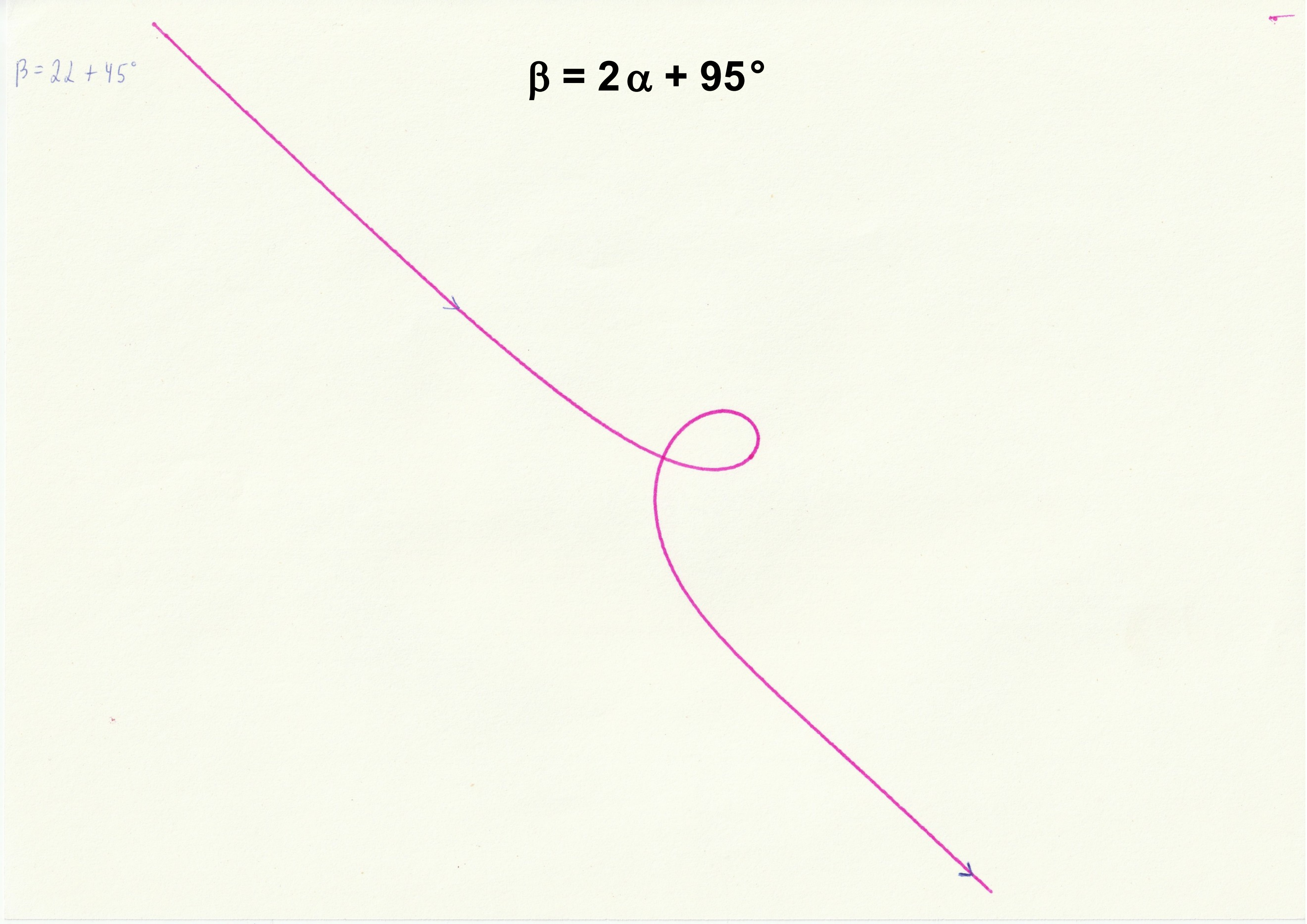
This W = 0 type curve on this and the previous page (both a circle) indicate that $V \rightarrow V' = 1 / V$ results in the same type of curve but they are spatially shifted

$\beta = 2\alpha + 0.5°$

$\beta = 2\alpha + 0.5°$

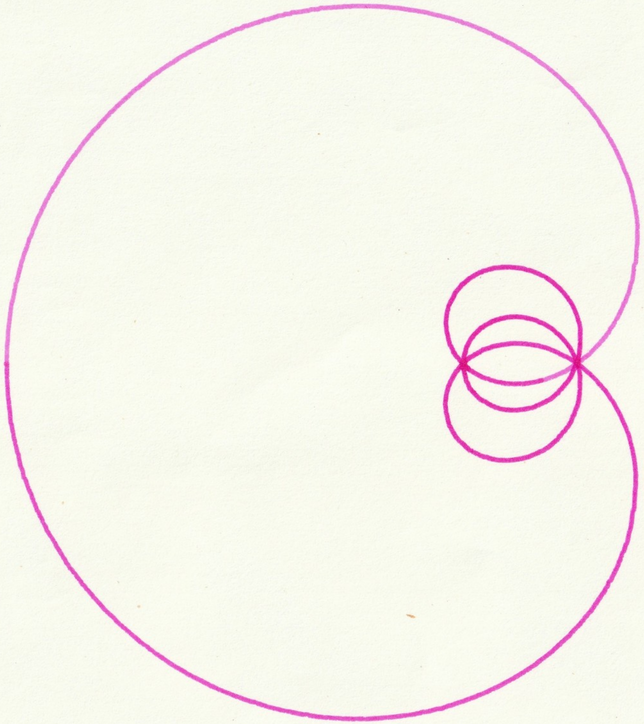


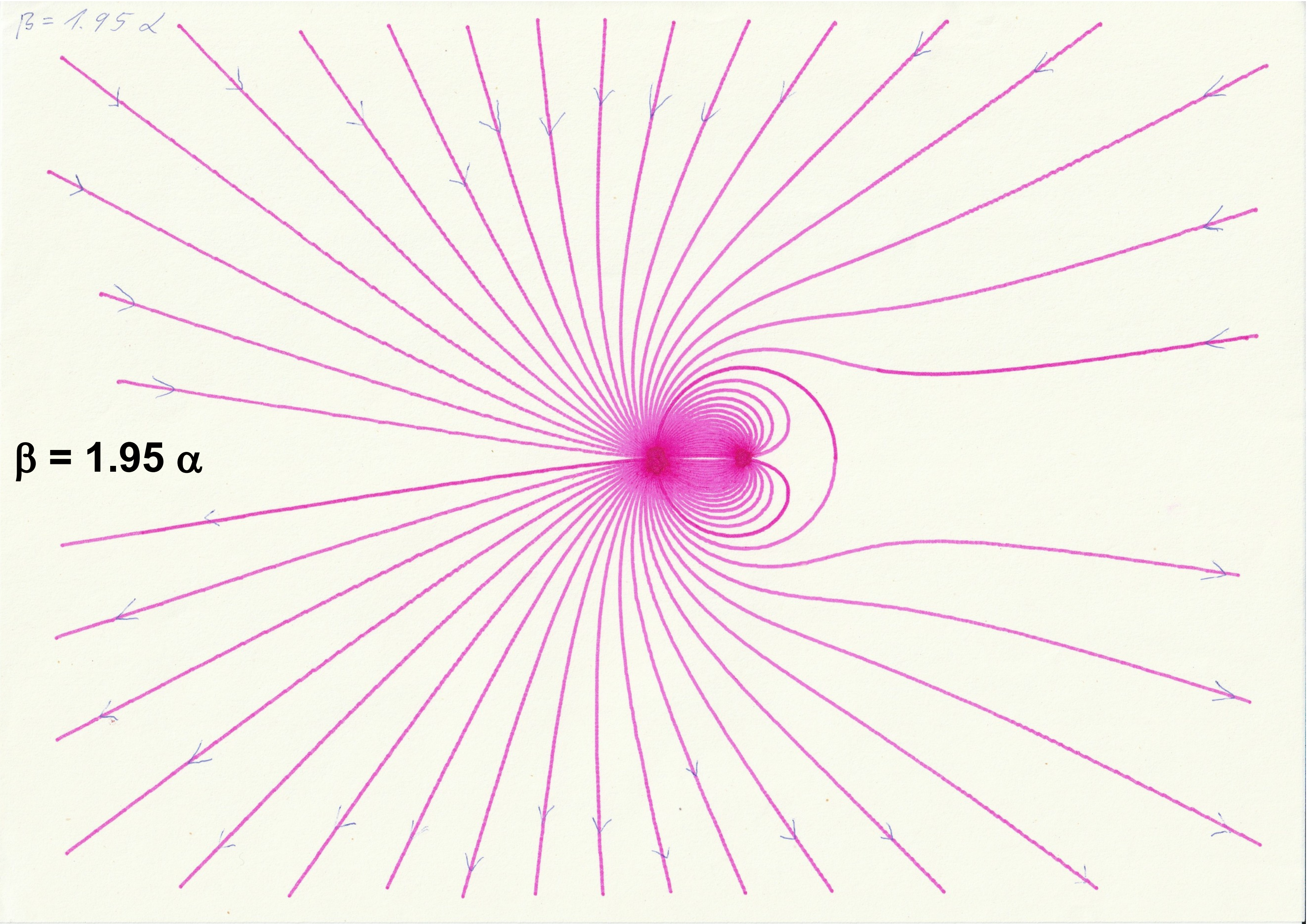
$\beta = 2\alpha + 95°$



$\beta = 0.8\alpha$

$\beta = 0.8\ \alpha$

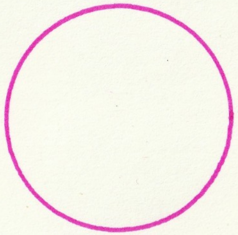




β = 1.95 α

$\beta = 2\alpha$

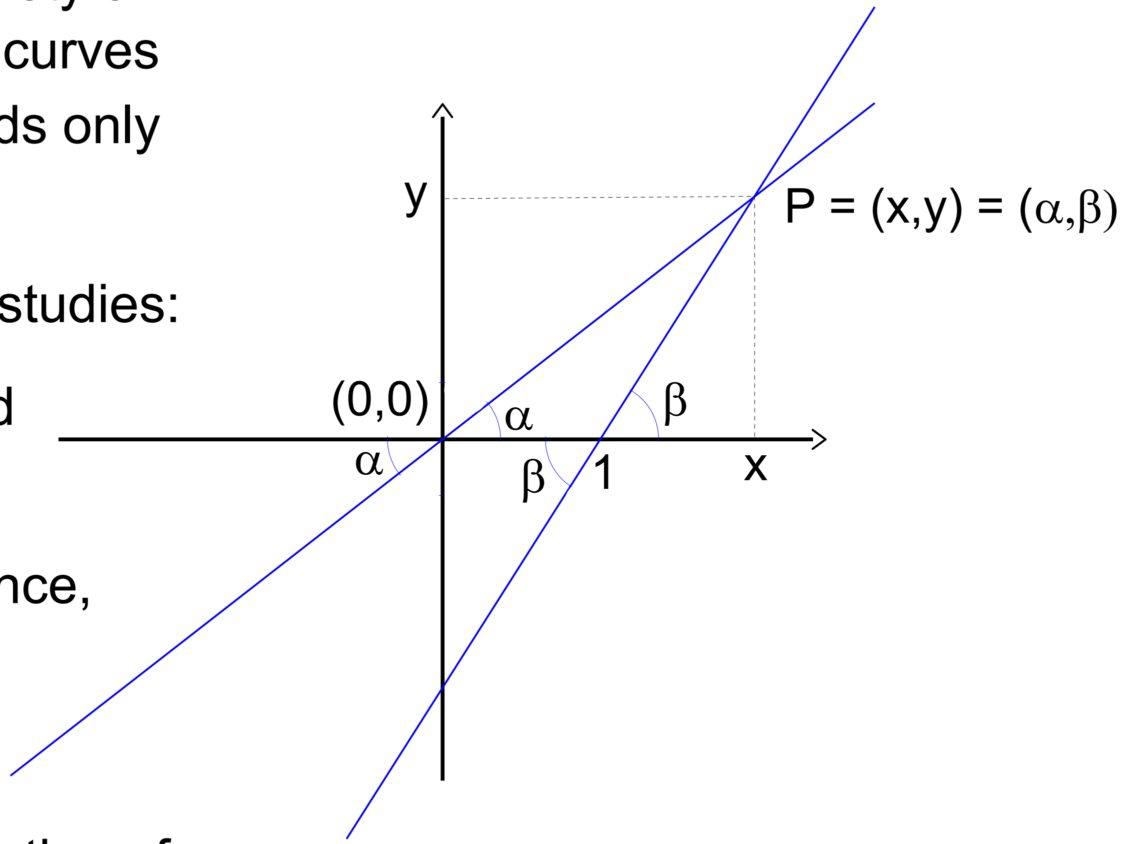
$\beta = 2\ \alpha$



3 Discussion / Open questions / Outlook ...

Discussion / Open questions / Outlook

It appears remarkable that such a variety of interesting, nice, and organic-looking curves are already generated when β depends only linearly from α , i.e. $\beta = V\alpha + W$

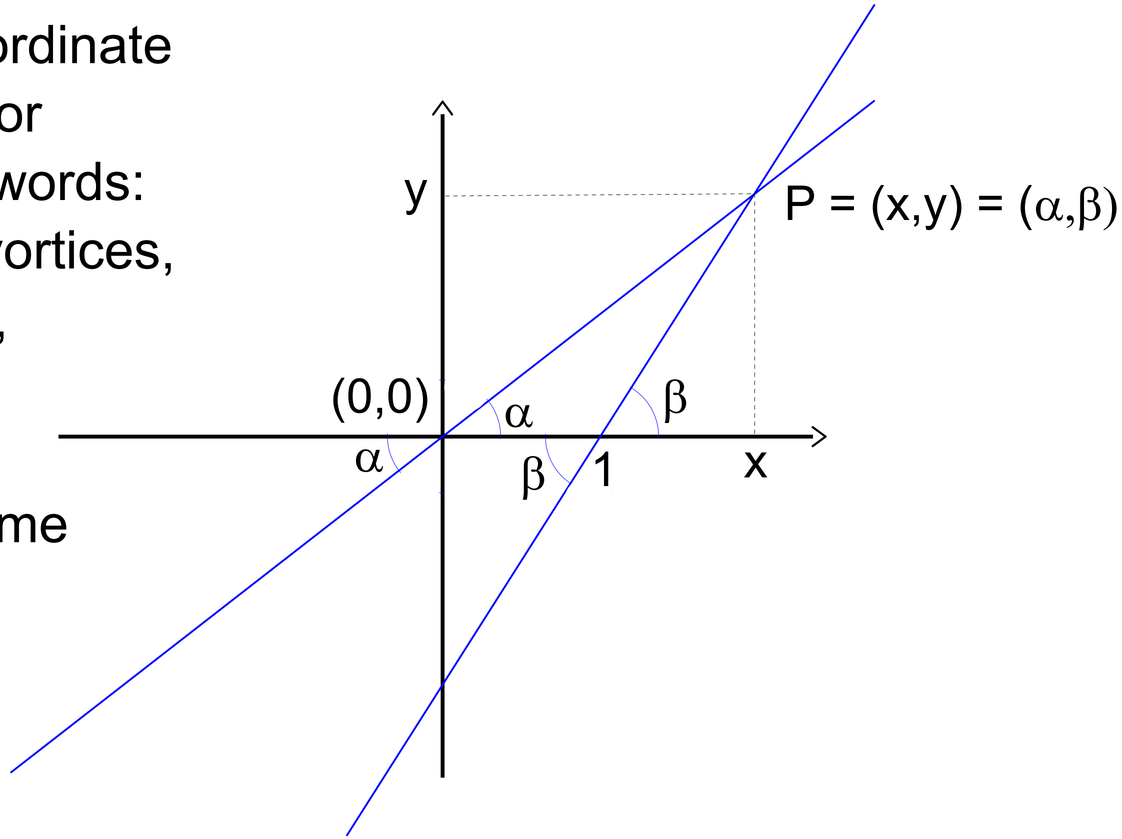


Open questions and potential further studies:

- Explore curves which are generated by a non-linear function $\beta = \beta(\alpha)$
- Are there some phenomena in science, physics, and metaphysics where a pure angular-based coordinate system provides a better understanding / description or a solution of the underlying mathematical equations ?

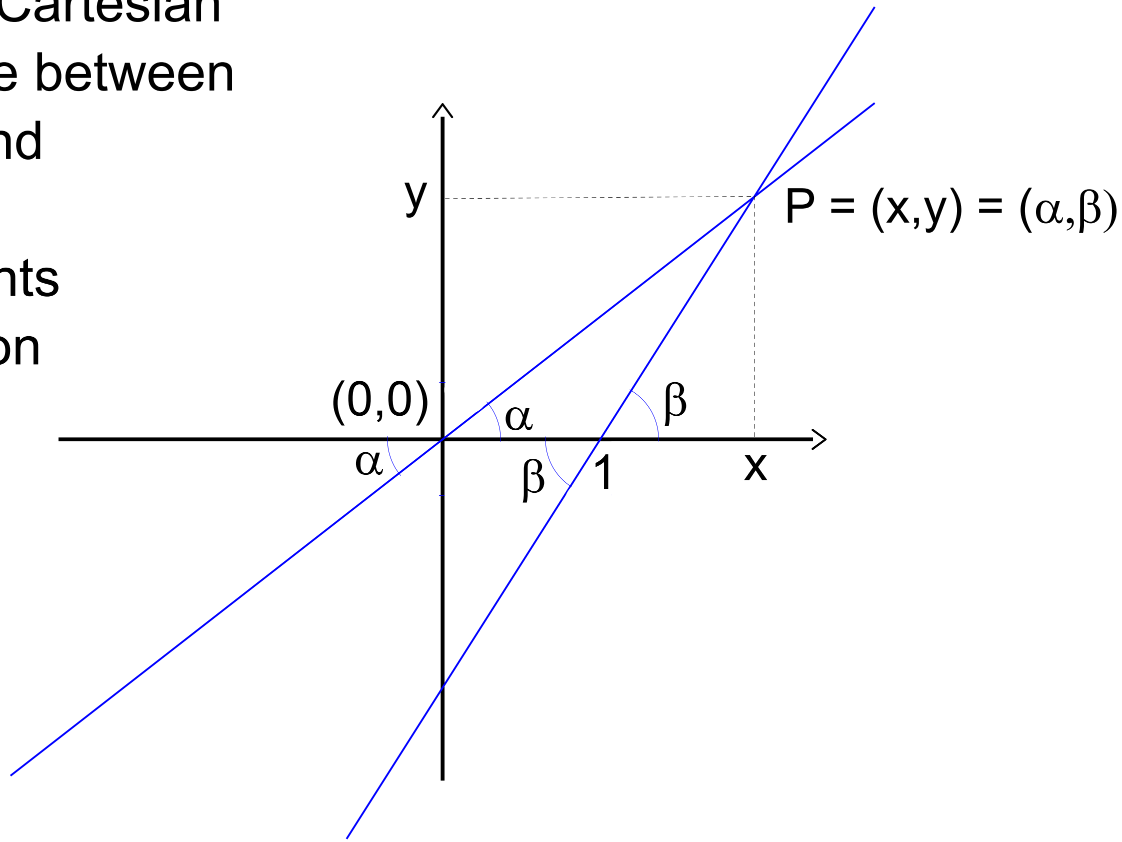
Discussion / Open questions / Outlook

- Explore if a pure angular-based coordinate system is potentially useful for one or several of the following topics / keywords:
  Angular momentum, spin, torsion, vortices, vortex physics, projective geometry,
  three-body problem,
  N-body problem,
  curvature / torsion / spin of space-time
- Extension of a pure angular-based coordinate system to three
  and even more
  (geometrical or spatial) dimensions



Discussion / Open questions / Outlook

In our used definition of α and β in a Cartesian coordinate system the connection line between the two pivot positions (x,y) = (0,0) and (x,y) = (1,0) is the x-axis. A potential disadvantage of this setup is that points P which are located on that connection line or the x-axis cannot be defined by α and β . That can be avoided for example by a line or ray whose pivot position is not located in the xy-plane but in the third dimension, i.e. at a position perpendicular to the drawing plane. That is illustrated on the following three pages ...



Discussion / Open questions / Outlook

This picture of a three-dimensional Cartesian coordinate system is used to create an illustration which is presented on the following two pages ...

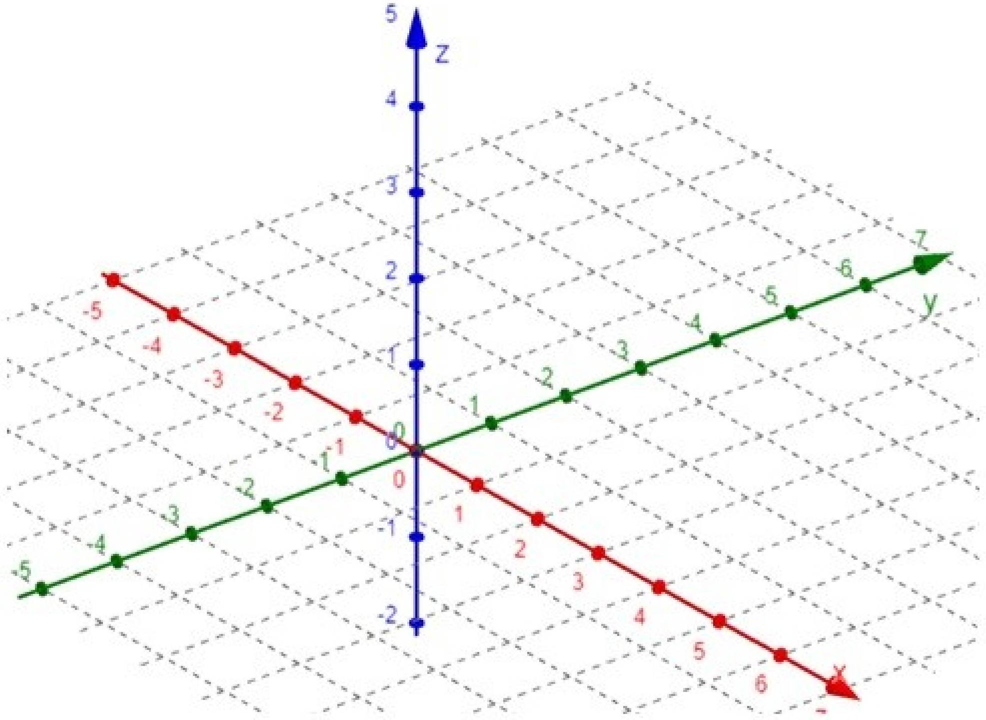
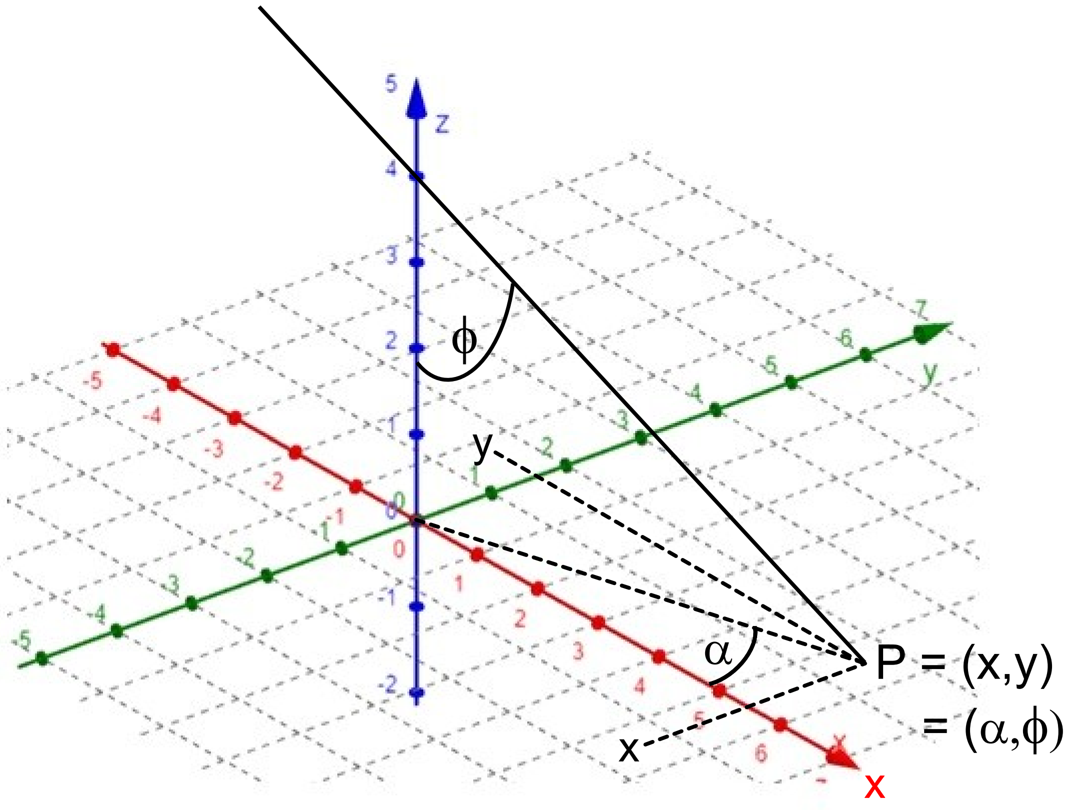


Image source:
https://www.tutorialspoint.com/computer_graphics/computer_graphics_modelling_3d_coordinate_systems.htm

Discussion / Open questions / Outlook

Sketch of an example of a pure angular-based coordinate system which defines the position of a point P in the two-dimensional xy-plane by using a line or ray whose pivot is located in the third dimension at (x,y,z) = (0,0,4).

A point P = (x,y) = (α,ϕ) in the xy-plane is here defined as the intersection of that line or ray and the xy-plane. An example of the definition of the the angles α and ϕ is indicated. Note that the angle ϕ is located outside of the xy-plane.



Discussion / Open questions / Outlook

It would be interesting to study which types of curves are created in the xy-plane when ϕ is a function of α , i.e.

$\phi = \phi(\alpha)$

or in the form of a parameter representation

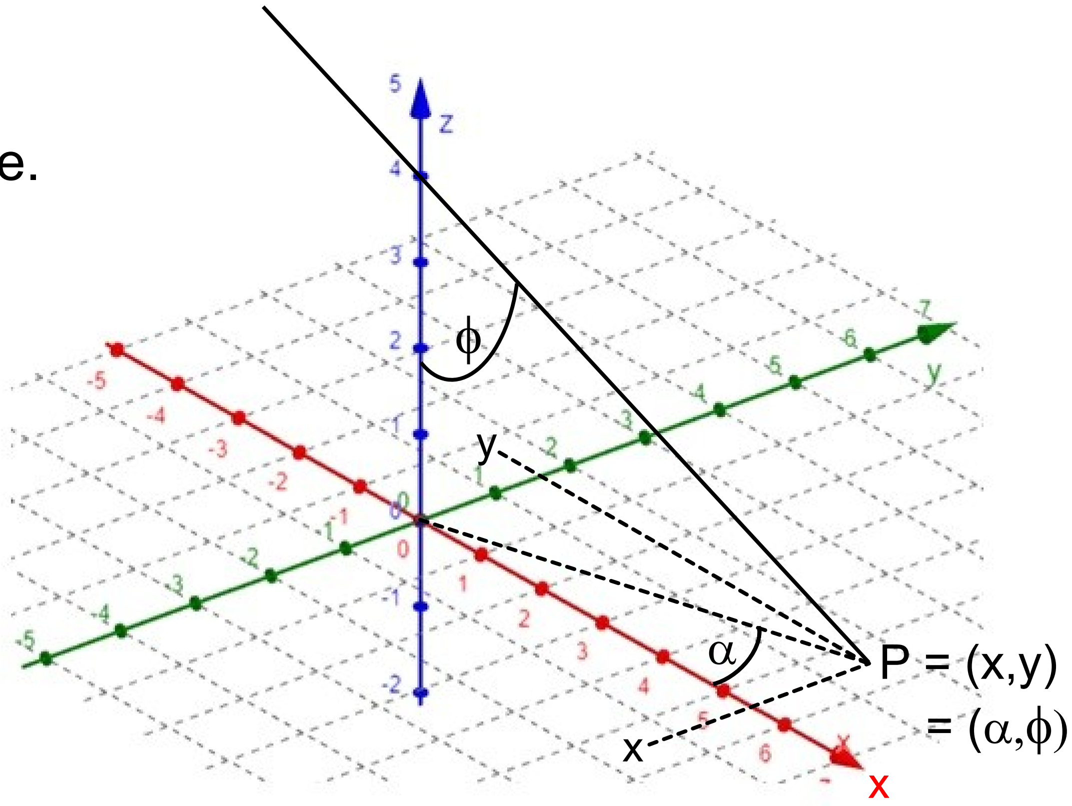
$\alpha = \alpha(t)$

$\phi = \phi(t)$

whereby t is a progressive parameter. One could study first the results of a linear relation

$\phi = V\alpha + W$

where V and W are parameters.



4 A related paper by Claude Ziad Bayeh from 2012 ...

In 2026, during the preparation period of this paper / presentation, the author discovered on the internet the following related paper by Claude Ziad Bayeh from 2012:

Introduction to the Angular Coordinate System in 2D space

Claude Ziad Bayeh

International Journal of Mathematical Models and Methods in Applied Sciences, Issue 4, Volume 6, 2012, pages 560 - 574

https://www.naun.org/main/NAUN/ijmmas/16-211.pdf